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Neutron Interferometry in Gravitational Field with Torsion

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We consider the possibility of finding experimental evidence of the fifth force with the measurement of a phase shift of neutron beams via an interferometric apparatus and also a possible rotation of the polarization plane of polarized neutron beams when torsion is introduced in a gravitational field.

1. INTRODUCTION

A large number of theoretical models and experiments claim the existence of new intermediate-range forces. Results of the experiments (Stacey *et al.*, 1987; Thierberger, 1987; Stubbs *et al.*, 1987; Niebauer *et al.*, 1987; Boynton *et al.*, 1987) and phenomenological considerations suggest that there might be a deviation from the Newtonian inverse square law with a potential of the form

$$V(r) = -G_\infty(M/r)(1+\Delta V) \tag{1}$$

Here $\Delta V(r)$ is caused by existence of a new type of interaction called the "fifth force."

Evidently there are several questions. The most important of them are:

(a) If the "fifth force" exists, to what attribute of matter could it couple?
(b) What is (are) the mediating particle (particles) of the "fifth interaction"?

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(c) What types of new effects both in classical and quantum regions can be predicted in the presence of the new interaction?

At present there are different opinions on the possible existence and nature of a "fifth force" and there are numerous classes of unified field models in which the new types of interactions can be predicted. In this paper we consider the quantum treatment of the fifth force due to the exchange of tordions in the frame of gauge gravitational theory, and regard two possible ways for experimental verification of this model of the fifth interaction. In the next section we give a phenomenological description of the neutron interferometry scheme. In Section 3 we propose models for fifth force mediators, i.e., mediating particles. Finally, in the last section we consider a neutron interferometry effect due to the interaction of neutrons with a "tordion field."

2. FIFTH FORCE AND NEUTRON INTERFEROMETRY

The fifth force is usually assumed to have a potential of the form (modification of the Newton law)

$$V(r) = -G_{\infty}(M/r)[1 + \alpha \exp(-r/\lambda)] \tag{2}$$

The potential energy due to gravity and an extra Yukawa interaction between two macroscopic bodies is given by

$$V_E(r) = -\int dr_1\, dr_2\, [G_{\infty}\rho(r_1)\rho(r_2)/r_{12}][1 + \alpha \exp(-r_{12}/\lambda)] \tag{3}$$

where ρ_1 and ρ_2 are mass densities, α is the strength of coupling of the fifth force, and λ is the range.

The energy contribution of the fifth force to the potential energy of a single neutron in the Earth's gravitational field is given by integration over the distribution of mass density acting on the neutron as

$$V_F(r) = (G_{\infty}E_n\alpha/c^2)\int d^3r'\, [\rho(r')/|\bar{r} - \bar{r}'|] \exp(-|\bar{r} - \bar{r}'|/\lambda) \tag{4}$$

where E_n is the neutron energy. For nonrelativistic neutrons $E_n/c^2 = m_n$ is the neutron rest mass. Integrating over the Earth volume and approximately assuming a uniform density, $\rho = 3M_{\oplus}/4\pi R_{\oplus}^3$, where $M_{\oplus}$ and $R_{\oplus}$ are, respectively, the mass and radius of the Earth.

Assuming $\lambda/R_{\oplus} \ll 1$ (as $\lambda \approx 200\ \text{m} \div 1\ \text{km} \ll R_{\oplus} \simeq 10^4\ \text{km}$), the above formula gives the additional potential energy contribution due to the fifth force acting on a neutron as (a neutron at the Earth's surface $r \simeq R_\oplus$)

$$V_F(R_\oplus) = (E_n/c^2)\alpha(G_\infty M_\oplus/R_\oplus)(\lambda/R_\oplus)^2 \tag{5}$$

So for neutrons of different energies E_{n_1} and E_{n_2} the energy difference is

$$\Delta V_F(R_\oplus) = (E_{n_1} - E_{n_2}/c^2)\alpha(GM_\oplus/R_\oplus)(\lambda/R_\oplus)^2 \tag{6}$$

For 20- and 10-GeV neutrons we find, with $\alpha \simeq 10^{-3}$, $\lambda \simeq 200$ m, $R_\oplus \simeq 6.4 \times 10^8$ cm, and $\alpha(\lambda/R_\oplus)^2 \simeq 9 \times 10^{-13} \simeq 10^{-12}$

$$\Delta V(R_\oplus) \approx 10^{-14}\ \text{eV} \tag{7}$$

For neutrons at rest or nonrelativistic neutrons

$$V_F(R_\oplus) \simeq 10^{-3}\alpha(\lambda/R_\oplus)^2 \simeq 10^{-15}\ \text{eV} \tag{8}$$

Now let us consider the phase shift on a neutron in a neutron interferometer. A gravitational phase shift was found in experiment (Colella *et al.*, 1975) and it is

$$|\Delta\Phi_G| = 4\pi(m/\hbar)(g/2v)A \tag{9}$$

where m is the gravitational mass, g is the acceleration due to gravity, v is the velocity of neutrons moving in the field, and A is the area enclosed by the interferometer beam paths. The explicit expression for $|\Delta\Phi_G|$ is

$$|\Delta\Phi_G| = |\Delta\Phi_0|[1 + (1/2)(gH/c^2)] \tag{10}$$

where

$$|\Delta\Phi_0| = 4\pi(m_0/\hbar)(g/2v)A \tag{11}$$

(with m_0 the rest mass of neutron) and gH/c^2 is the gravitational potential energy and H is the height difference of the paths. In general

$$m = m_0(1 + 2gH/c^2)(1 + 2gH/c^2 - v^2/c^2)^{-1/2} \tag{12}$$

and we can consider the relativistic effects.

In the presence of the fifth force we have an additional contribution to the potential energy of the neutron, i.e., to gH/c^2, i.e.,

$$gH/c^2 + V_F(R_\oplus) \quad \text{or} \quad \Delta V_F(R_\oplus) \tag{13}$$

in case of different energies. Then the phase shift is

$$|\Delta\Phi_{\text{tot}}| = \Delta\Phi_0[1 + 1/2(gH/c^2) \pm (E_n/c^2)\alpha(GM_\oplus/R_\oplus)(\lambda/R_\oplus)^2] \tag{14}$$

Evidently the fifth force effect will be smaller then the gravitational one. But there is one principal and important discrepancy. If we have horizontal parallel beams or, better, if we think of some experiment in a satellite, then $H=0$ and we have a net result for the fifth force term

$$|\Delta\Phi_5| = \pm\Delta\Phi_0(E_n/c^2)\alpha(G_\infty M_\oplus \lambda^2/R_\oplus^3) \tag{15}$$

We can see that the phase shift will take place in this case in the horizontal position of the interferometer.

Let us consider the most general potential for the fifth force (Talmadge and Fischbach, 1988), introducing also a composition dependence, but considering only the first term in the potential and only the baryon-dependent part:

$$V(r) = -(G_\infty M/r)[1 + \alpha_0 \exp(-r/\lambda_0) - \xi_0 q_B \exp(-r/\lambda_0)] \tag{16}$$

where α_0 and ξ_0 are constants which respectively describe the universal and nonuniversal (i.e., composition-dependent) contributions and $q_B = B/\mu$ (with B the baryon number and μ the atomic mass of the body).

After a calculation analogous to that performed above, we obtain the formula for the phase shift (we suppose that $H=0$):

$$|\Delta\Phi_5| = \Delta\Phi_0[1 \pm (E_n/c^2)(GM_\oplus/R^3)(\alpha_0\lambda_0 - \xi_0 q_B \lambda_0)] \tag{17}$$

Now we can perform a "null" experiment to verify the existence of the fifth force. If $|\Delta\Phi_5| - |\Delta\Phi_0| = 0$, it is only an indication of the equality of two exchanges due to scalar massive particles and due to vector particles. If $|\Delta\Phi_5| - |\Delta\Phi_0| <$ or >0, we can estimate the fifth force effect.

An extremely important question is the problem of the mediating particles of the fifth force (de Sabbata and Sivaram, 1990).

3. GAUGE GRAVITY WITH TORSION

Now let us consider gravitational theory, namely the gauge treatment of gravity in the framework of the Poincaré group, which allows us to understand the fifth force problem without an additional supposition of supersymmetry, scalar-tensor theory, and so on. The dynamical variables in the Poincaré gauge gravitational theory will be the metric $g_{\mu\nu}$ and affine connection, which satisfy the metricity condition $g_{\mu\nu;\alpha} = 0$ (Ivanenko *et al.*, 1985; Ponomariev *et al.*, 1985; de Sabbata and Gasperini, 1985). The base manifold for the Poincaré gauge gravitational theory will be the Riemann–Cartan space-time or U_4.

The most general gravitational Lagrangian includes more than 150 terms, which are second degree in curvature and fourth degree in the torsion field. The curvature and torsion are the field strengths for the local Lorentz connection and vierbeins. We do not need to use such a complicated gravitational Lagrangian to demonstrate the principal possibility of the origin of the

Yukawa type of interaction of fermions with gravity. We restrict ourselves to the action:

$$S=\int d^4x\,(-g)^{1/2}[(1/16\pi G_\infty)R(g,Q)-(1/4)(Q_{\mu,\nu}-Q_{\nu,\mu})^2$$
$$-\bar{\psi}_i\gamma^\mu(\nabla_\mu-\xi\gamma_5 Q_\mu)\psi+m\bar{\psi}\psi] \tag{18}$$

where $Q_\mu=1/2\varepsilon_{\mu\nu\alpha\beta}Q^{\nu\alpha\beta}$ is the pseudotrace of torsion, and ξ is the unknown spin-torsion constant of interaction.[5] The exact spherically symmetrical solutions of field equations in Poincaré gauge theory (without matter) were obtained in Camenzind (1975), Kim and Yoon (1987), and Ponomariev *et al.* (1990). But we would like to consider the quantum nature of the gravitational potential.

We regard the weak field limit and suppose that the metric and torsion are satisfied up to the next gauge fixing terms $\partial_\mu h^{\nu\mu}=0$ and $\partial_\alpha Q^\alpha=0$. Then the Lagrangian takes the form

$$\mathscr{L}=h^{\mu\nu}P_{\mu\nu\alpha\beta}\,\Box h^{\alpha\beta}+Q_\alpha(\Box+m_q^2)Q^\alpha-k^2h^{\mu\nu}T_{\mu\nu}+\xi S_\mu Q^\mu \tag{19}$$

where $P_{\mu\nu\alpha\beta}=\frac{1}{4}(\eta_{\mu\alpha}\eta_{\nu\beta}+\eta_{\nu\alpha}\eta_{\mu\beta}-\eta_{\mu\nu}\eta_{\alpha\beta})$, $\Box=\partial^\alpha\partial_\alpha$, $T_{\mu\nu}$ is the energy-momentum tensor

$$T_{\mu\nu}=(i/2)[\bar{\psi}\gamma_{(\mu}\partial_{\nu)}\psi-\partial_{(\mu}\bar{\psi}\gamma_{\nu)}\psi]-\eta_{\mu\nu}\mathscr{L}(\psi,Q=0,g=\eta)$$

S^μ is the vector of spin

$$S_\mu=i\bar{\psi}\gamma_\mu\gamma_5\psi$$

and m_q is the mass of the mediating particle.

The Green's functions for gravitons and tordions (the particles of the torsion field) are

$$D_{\mu\nu\alpha\beta}=P_{\mu\nu\alpha\beta}\left(\frac{1}{2\pi}\right)^4\int d^4k\,\frac{e^{-ikx}}{k^2+i\varepsilon} \tag{20a}$$

$$D_{\mu\nu}=\frac{1}{2\pi}\int dk\,\frac{e^{-ikx}}{k^2+m_q^2+i\varepsilon} \tag{20b}$$

The potential can be calculated by means of Gupta's (1977) method. We omit here the details of the calculation and give the final result for the

[5]We can introduce two independent constants in Poincaré gauge theory because there are two normal subgroups in $ISO(3,1)=SO(3,1)\times T(4)$. For a general discussion on the torsion coupling constant see de Sabbata and Sivaram (1989).

potential form, which is

$$V_{\text{tot}} = V_0 + \Delta V \tag{21}$$

where V_0 is the well-known potential which is caused by graviton exchange, and

$$\Delta V = \frac{1}{2}\xi^2 \frac{G_\infty}{m_q^2}\left[-m_q^2 2\frac{\bar{\sigma}_1 \cdot \bar{\sigma}_2}{r}\exp(-2^{1/2}m_q r) + (\bar{\sigma}_1 \cdot \bar{\nabla})(\bar{\sigma}_2 \cdot \bar{\nabla})\frac{\exp(-2^{1/2}m_q r)}{r}\right] \tag{22}$$

is the potential due to tordion exchange. Here $\bar{\sigma}_1$, $\bar{\sigma}_2$ are the Pauli matrices for pairs of fermions.

So it is clear that the interaction between two neutrons due to graviton and tordion exchanges leads to a gravitational potential exactly of the form (1) in the static limit.

We would like to underline that in the proposed model we have no need of additional fields besides the gravitational field, because the latter is characterized by two independent dynamical variables: metric and torsion see also (de Sabbata and Gasperini, 1985; de Sabbata and Sivaram, 1990; de Sabbata *et al.*, 1991; Ivanenko *et al.*, 1985; Ponomariev *et al.*, 1985). The torsion field interacts with spins of particles and the neutron fields will have an additional phase shift due to this interaction.

4. EFFECTS OF TORSION

We can consider the special effect of torsion on the interferometer. In that case we may have an additional term in energy like $3G^2S^2/c^4R^4$, so in an experimental arrangement in which $H=0$, we have a net result ($gH/c^2 = 0$, $\xi = 1/16\pi G$)

$$|\Delta\Phi_5| = |\Delta\Phi_0|(1 \pm 3GS^2/c^4R_\oplus^4) \tag{23}$$

But in the interaction of torsion with spins we have a principally new picture, namely, polarization plane rotation (de Sabbata and Gasperini, 1982; de Sabbata and Sivaram, 1989*a*; Pronin, 1987). Now let us consider some details of interferometry with polarized neutrons in an external torsion field.

To separate the spin-torsion interaction effect, we restrict consideration to Minkowski–Cartan space, supposing that $Q_\mu = \{0, \mathbf{Q}\}$, where $\mathbf{Q}$ is a constant vector, $g_{\mu\nu}=\eta_{\mu\nu}=\mathrm{diag}(1, -1, -1, -1)$. Then the Pauli equation for spinors is

$$i\hbar\frac{\partial\psi}{\partial t}=\left[-\frac{\hbar^2}{2m}\bar{\nabla}^2+\xi(\bar{\sigma}\cdot\bar{Q})\right]\psi+\cdots \tag{24}$$

We assume that the neutron beams (I and II) are polarized in the antiparallel direction to the z axis. Then the spinor normalized functions are

$$\psi_{\mathrm{I}}=|\downarrow_z\rangle=\begin{bmatrix}0\\1\end{bmatrix}, \qquad \psi_{\mathrm{II}}=e^{i\vartheta}|\downarrow_z\rangle=e^{i\vartheta}\begin{bmatrix}0\\1\end{bmatrix} \tag{25}$$

where ϑ is the phase shift.

If the external torsion field equals zero, the degree of polarization of the beam after interferometry is

$$\bar{P}=\frac{\psi_f^+\bar{\sigma}\psi_f}{\psi_f^+\psi_f}=\{0, 0, -1\} \tag{26}$$

where $\psi_f=\psi_{\mathrm{I}}+\psi_{\mathrm{II}}$.

But in the situation when one (or both) neutron beam(s) interacts with torsion, the states ψ_{I} and ψ_{II} should be changed as

$$\begin{aligned}\psi'_{\mathrm{I}}&=\varepsilon^{1/2}|\uparrow_z\rangle+(1-\varepsilon)^{1/2}|\downarrow_z\rangle,\\ \psi'_{\mathrm{II}}&=e^{-i\vartheta}\{\varepsilon^{1/2}|\uparrow_z\rangle+(1-\varepsilon)^{1/2}|\downarrow_z\rangle\}\end{aligned} \tag{27}$$

Then the integration over these beams leads to

$$\bar{P}'=\frac{\psi_f^{+\prime}\bar{\sigma}\psi'_f}{\psi_f^{+\prime}\psi'_f}=\{2[\varepsilon(1-\varepsilon)]^{1/2}, 0, -1+2\varepsilon\} \tag{28}$$

and here $\psi'_f=\psi'_{\mathrm{I}}+\psi'_{\mathrm{II}}$.

For example, when $\varepsilon=1/2$, we have $\bar{P}'=\{1, 0, 0\}$.

So we can observe the effect of the polarized rotation plane due to quantum interferometry, which is caused by the previous interaction with torsion.

Let us give some estimation of the effect: we consider $\varepsilon\simeq\xi|\bar{Q}|$, where $|\bar{Q}|$ is proportional to polarized particles density n. It is well known that $|\bar{Q}|=\hbar cn$. In laboratory experiments it will be convenient to use the polarized particle beams in the accelerator. Then in the optimal situation ($\varepsilon=1/2$) we can find the upper limit on the spin-torsion interaction constant, because

$$\xi\leq 10^{17}/2n \tag{29}$$

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